CERTAIN CASES OF THE ELASTIC EQUILIBRIUM OF AN INFINITE WEDGE WITH A NONSYMMETRIC NOTCH AT THE VERTEX, SUBJECTED TO CONCENTRATED FORCES

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An effective form of the solution is proposed for a Hilbert problem relative to the two-dimensional vector function, which is convenient for applications. Cases of practical importance, of the elastic equilibrium of an infinite triangular wedge weakened by a nonsymmetric notch at the vertex, reduce to the mentioned Hilbert problem which was considered earlier in [1]. The solution of the first fundamental problem is elucidated herein for three cases:

1) A notch on the continuation of one side of the wedge;
2) A notch issuing on the boundary of the half-plane;
3) A semi-infinite slit with a breakpoint in the infinite plane.

Formulas are presented for the coefficient of stress intensity near the aperture of the wedge, and for the dislocations of its sides under the effect of equal and opposite concentrated forces.

1. Canonical matrix of one homogeneous Hilbert problem for a two-dimensional vector function. Let the complex z plane be divided by a simple closed contour L into two domains: Ω^+ (outside L) and Ω^- (within L). Furthermore, let a square second order matrix $X(z)$ be holomorphic together with its inverse in the domains Ω^+ and Ω^- everywhere except at a finite number of points (poles), and let it have finite limits $X^+(t)$ and $X^-(t)$, for $t \in L$ if the point z approaches L from the right while remaining in Ω^+, or from the left while remaining in Ω^-.

Let us examine the homogeneous equation [2, 3]

$$X^+(t)\,[X^-(t)]^{-1} = G(t) \tag{1.1}$$

where $G(t)$ is a 2 x 2 matrix given on L, which is nowhere singular and satisfies the Hölder condition. The matrix $X(z)$ possessing the above-mentioned properties and satisfying (1.1) on L is a rational solution of this equation. The canonical solution of the problem (1.1) i.e., the solution in which the matrices $X(z)$ and $X^{-1}(z)$ are piecewise-holomorphic, and the order of the determinant of the matrix $X(z)$ at infinity is the sum of the order of the column-vectors, can be obtained effectively from the rational solution [1, 4]. The problem of seeking a class of matrices $G(t)$ admitting solution of (1.1) was formulated in [4] under the additional condition

$$X^+(t)\,[X^-(t)]^{-1} = [X^-(t)]^{-1}\,X^+(t) \tag{1.2}$$

This problem was solved in [1] by obtaining (to the accuracy of a matrix factor holomorphic in Ω^+ or Ω^-)

$$G(t)=b(t)\begin{Vmatrix}1 & 0\\ 0 & 1\end{Vmatrix}+c(t)\begin{Vmatrix}l(t) & m(t)\\ n(t) & -l(t)\end{Vmatrix} \tag{1.3}$$

where $b(t)$ and $c(t)$ are arbitrary functions, $l(t)$, $m(t)$ and $n(t)$ are polynomials, and special constraints are still imposed on the matrix function $\lg G(t)$. The problem (1.1) for the case when the matrix $G(t)$ has the form (1.3) underlies the subsequent exposition, hence a solution of this problem is given below in a simple form convenient for analysis.

One of the following propositions

$$l^2+mn\equiv 0; \tag{1.4}$$

$$l^2+mn=[g(z)]^2 f(z) \tag{1.5}$$

is valid relative to the polynomial $(l^2+mn)(z)$ where $g(z)$, $f(z)$ are polynomials, and the set of zeroes of $f(z)$ is formed by the zeroes of odd multiplicity of the polynomial $(l^2+mn)(z)$ each taken once.

In the case (1.4) we seek the solution in the form

$$\mathrm{X}(z)=\zeta(z)\begin{Vmatrix}1 & 0\\ 0 & 1\end{Vmatrix}+\omega(z)\begin{Vmatrix}l(z) & m(z)\\ n(z) & -l(z)\end{Vmatrix} \tag{1.6}$$

$$\mathrm{X}^{-1}(z)=[\zeta(z)]^{-1}\begin{Vmatrix}1 & 0\\ 0 & 1\end{Vmatrix}-[\zeta(z)]^{-2}\omega(z)\begin{Vmatrix}l(z) & m(z)\\ n(z) & -l(z)\end{Vmatrix} \tag{1.7}$$

The functions $\zeta(z)$ and $\omega(z)$ are to be determined here. Condition (1.2) is hence satisfied identically, and (1.1) is satisfied if the two relationships

$$\zeta^+(t)\,[\zeta^-(t)]^{-1}=b(t) \tag{1.8}$$

$$\omega^+(t)\,[\zeta^+(t)]^{-1}-\omega^-(t)\,[\zeta^-(t)]^{-1}=c(t)\,[b(t)]^{-1} \tag{1.9}$$

are satisfied on L

Since $\det G(t)\neq 0$, then under condition (1.4), $b(t)\neq 0$ also. It hence follows that (1.8) has a solution in the form of the piecewise holomorphic functions $\zeta(z)$, which never vanish at a finite distance. Then (1.9) also has a solution in the form of the piecewise holomorphic function $\omega(z)$. Therefore, not only a rational but also a normal (Without poles at a finite distance) solution of the problem (1.1) satisfying condition (1.2) is given by (1.6), (1.7).

Let us introduce a special form for the square, second-order matrix A Let a matrix of the form

$$\operatorname{dev} A=A-sI$$

be called the deviator of the 2x2 matrix A, where s is the half-trace of the matrix A and I is the unit 2x2 matrix. Let A be a matrix with nonsingular deviator. Let us write

$$(\det A)^{-1/2}A=(\det A)^{-1/2}sI+(\det A)^{-1/2}(-\det\operatorname{dev} A)^{1/2}(-\det\operatorname{dev} A)^{-1/2}\operatorname{dev} A \tag{1.10}$$

Taking into account the identity

$$s^2+\det\operatorname{dev} A=\det A$$

we give the relationship (1.10) the form

$$(\det A)^{-1/2}A=I\operatorname{ch}\varepsilon+(-\det\operatorname{dev} A)^{-1/2}\operatorname{dev} A\operatorname{sh}\varepsilon \tag{1.11}$$

$$\varepsilon = \ln \frac{s + (-\det \operatorname{dev} A)^{1/2}}{(\det A)^{1/2}} = \frac{1}{2} \ln \frac{s + (-\det \operatorname{dev} A)^{1/2}}{s - (-\det \operatorname{dev} A)^{1/2}} = \frac{1}{2} \ln \frac{\lambda_1}{\lambda_2}$$

Here $\lambda_{1,2}$ are the eigennumbers of the matrix A. We call the parameter ε the index of the matrix A.

We call the matrix

$$\operatorname{com} A = (-\det \operatorname{dev} A)^{-1/2} \operatorname{dev} A, \qquad (\operatorname{com} A)^2 = I$$

the commutant of the matrix A. We obtain

$$A = (\det A)^{1/2} (I \operatorname{ch} \varepsilon + \operatorname{com} A \operatorname{sh} \varepsilon) \tag{1.12}$$

The following multiplication rule for second-order matrices represented in the special form (1.12) and having a common commutant is verified directly. In order to multiply two such matrices it is sufficient to multiply their determinants and to add the indices while conserving the common commutant (in particular, there hence results that the considered matrices mutually commutate). To invert a matrix in the form (1.12) it is sufficient to invert the determinant and to change the sign of the index while conserving the commutant as before.

Turning to a consideration of the case (1.5), we initially assume that $g(z)$ does not vanish anywhere on L, and we represent the matrix $G(t)$ in the special form

$$\Delta^{-1/2} G(t) = I \operatorname{ch} \varepsilon(t) + B(t) \operatorname{sh} \varepsilon(t) \tag{1.13}$$

Here $\Delta(t)$, $\varepsilon(t)$ and $B(t) = \operatorname{com} G(t)$ are the determinant, index and commutant of the matrix $G(t)$, where $B(t)$ is the boundary value of a matrix of the form

$$B(z) = (l^2 + mn)^{-1/2} \begin{Vmatrix} l(z) & m(z) \\ n(z) & -l(z) \end{Vmatrix} \tag{1.14}$$

on L, which is defined everywhere in the z plane. We seek the solution in the form

$$\Lambda^{-1}(z) \mathrm{X}(z) = I \operatorname{ch} [(f^{1/2}\beta)(z)] + B(z) \operatorname{sh} [(f^{1/2}\beta)(z)] \tag{1.15}$$

$$\Lambda(z) \mathrm{X}^{-1}(z) = I \operatorname{ch} [(f^{1/2}\beta)(z)] - B(z) \operatorname{sh} [(f^{1/2}\beta)(z)] \tag{1.16}$$

where the functions $\Lambda(z)$ and $\beta(z)$ are to be determined. Condition (1.2) is hence satisfied identically, and (1.1) is satisfied if the relationships

$$\Lambda^+(t) [\Lambda^-(t)]^{-1} = \Delta^{1/2}(t), \quad \beta^+(t) - \beta^-(t) = f^{-1/2}(t) \, \varepsilon(t) \tag{1.17}$$

are satisfied on L

Let us use the notation

$$(4\pi i)^{-1} [\ln (\lambda_1 \lambda_2)] |_L = \varkappa_\Delta, \qquad (4\pi i)^{-1} \left[\ln \frac{\lambda_1}{\lambda_2}\right] \bigg|_L = \varkappa_\varepsilon \tag{1.18}$$

$$\varkappa_\Delta + \varkappa_\varepsilon = (2\pi i)^{-1} [\ln \lambda_1] |_L, \qquad \varkappa_\Delta - \varkappa_\varepsilon = (2\pi i)^{-1} [\ln \lambda_2] |_L$$

where $\lambda_{1,2}(t)$ are the characteristic functions (eigennumbers) of the matrix $G(t)$. Piecewise-holomorphic solutions of the problems (1.17) are given by the formulas ([2], p. 131)

$$\Lambda(z) = (z-a)^{\varkappa_\Delta} \exp\left[-\frac{1}{4\pi i}\int_L \frac{\ln\Delta(t)}{t-z}\,dt\right] \tag{1.19}$$

$$\beta(z) = f^{-1/2}(a)\,\varkappa_\varepsilon \ln(z-a) - \frac{1}{2\pi i}\int_L \frac{f^{-1/2}(t)\,\varepsilon(t)}{t-z}\,dt \tag{1.20}$$

Here a is an arbitrary point on L, at which the origin and terminus of the path of integration in (1.19) and (1.20) coincide.

For the solution defined by (1.15), (1.16) to have an integer bounded order at infinity, it is sufficient to set $\varkappa_\varepsilon = 0$ and if $k > 2$ (k is the degree of the polynomial $f(z)$), the following equalities

$$\int_L t^{\alpha-1} f^{-1/2}(t)\,\varepsilon(t)\,dt = 0 \qquad (\alpha = 1, \ldots, \gamma) \tag{1.21}$$

must still be satisfied on L, where γ is the greatest of the integers such that the quantity $2\gamma + 1$ does not exceed k. The quantity $\varkappa_\Delta$ is an integer, and the function $f^{1/2}(z)\,\beta(z)$ is defined by the relationship

$$f^{1/2}(z)\,\beta(z) = \frac{f^{1/2}(z)}{2\pi i}\int_L \frac{f^{-1/2}(t)\,\varepsilon(t)}{t-z}\,dt \tag{1.22}$$

whose right side is bounded at infinity by virtue of (1.21). The function $\Lambda(z)$ never vanishes at a finite distance. The presence of isolated singularities (poles) of the commutant $B(z)$ in the zeroes of the polynomial $g(z)$, although not taken into account in obtaining the form (1.12), is in complete conformity with the definition of a rational solution. If $g(z)$ has zeroes on L, then the corresponding solution which has poles on L can be obtained by a passage to the limit.

Let us note that the result obtained for (1.1) on a simply-connected closed contour remains valid even for a contour in the form of an infinite line (one of the coordinate axes, say), if the elements of the matrix $G(t)$ satisfy the Hölder condition in the neighborhood of a point at infinity.

2. Certain problems for an infinite wedge with a rectilinear nonsymmetric notch at the vertex. Let be given an infinite triangular wedge with a notch (0,1) on the line $\varphi = 0$, where the edges of the wedge $\varphi = \theta_1$ and $\varphi = -\theta_2$ are stress free, and prescribed loads, equal in magnitude and opposite in direction, are applied to the sides of the notch. Let us denote the components of the stress-strain state as follows:

$$\frac{\partial u_r}{\partial r} = g_1(r,\varphi), \quad \frac{\partial u_\varphi}{\partial r} = g_2(r,\varphi), \quad \tau_{r\varphi} = g_3(r,\varphi), \quad \sigma_\varphi = g_4(r,\varphi)$$

Let us denote the corresponding Mellin transforms as follows:

$$g_j^\circ(\varphi, p) = \int_0^\infty r^p g_j(r,\varphi)\,dr$$

Examining the first fundamental problem for the wedge $0 \leqslant \varphi \leqslant \theta_1$ [5], we obtain the following relationships between the transforms of the derived displacements

$g_{1,2}{}^{\circ}(+0, p)$ and the stresses $g_{3,4}{}^{\circ}(0, p)$ on the notch lines:

$$\begin{aligned}
{}^1/_2 E g_2{}^{\circ}(+0, p) &= D(p, \theta_1)\ (\sin p\theta_1 \cos p\theta_1 + p \sin \theta_1 \cos \theta_1)\, g_4{}^{\circ}(0, p) + \\
&+ [p(p-1)\sin^2\theta_1 D(p, \theta_1) - {}^1/_2(1-\nu)]\, g_3{}^{\circ}(0, p) \\
{}^1/_2 E g_1{}^{\circ}(+0, p) &= [-p(p+1)\sin^2\theta_1 D(p, \theta_1) + {}^1/_2(1-\nu)]\, g_4{}^{\circ}(0, p) + \\
&+ D(p, \theta_1)(\sin p\theta_1 \cos p\theta_1 - p \sin\theta_1 \cos\theta_1)\, g_3{}^{\circ}(0, p) \\
D(p, \theta) &= (p^2 \sin^2\theta - \sin^2 p\theta)^{-1}
\end{aligned} \tag{2.1}$$

where E, ν are the elastic modulus and Poisson's ratio, respectively. Analogous relationships hold for the wedge $0 \geqslant \varphi \geqslant -\theta_2$ but with $+0$ replaced by -0 and θ_1 by $-\theta_2$. Hence, the following expressions result for the discontinuities of the transforms of the derived displacements on the line $\varphi = 0$:

$$\begin{aligned}
&{}^1/_4 E[g_2{}^{\circ}(+0, p) - g_2{}^{\circ}(-0, p)] = {}^1/_2 p(p-1)[\sin^2\theta_1 D(p, \theta_1) - \\
&- \sin^2\theta_2 D(p, \theta_2)]\, g_3{}^{\circ}(0, p) + \\
&+ {}^1/_4[D(p, \theta_1)(\sin 2p\theta_1 + p \sin 2\theta_1) + D(p, \theta_2)(\sin 2p\theta_2 + p \sin 2\theta_2)]\, g_4{}^{\circ}(0, p) \\
&{}^1/_4 E[g_1{}^{\circ}(+0, p) - g_1{}^{\circ}(-0, p)] = {}^1/_2 p(p+1)[\sin^2\theta_2 D(p, \theta_2) - \\
&- \sin^2\theta_1 D(p, \theta_1)]\, g_4{}^{\circ}(0, p) + {}^1/_4[D(p, \theta_1)(\sin 2p\theta_1 - p \sin 2\theta_1) + \\
&+ D(p, \theta_2)(\sin 2p\theta_2 - p \sin 2\theta_2)]\, g_3{}^{\circ}(0, p)
\end{aligned} \tag{2.2}$$

Let us introduce the two-dimensional 2 x 1 column vector of the form

$$u(r) = \{{}^1/_4 E[g_2(r, +0) - g_2(r, -0)],\ {}^1/_4 E[g_1(r, +0) - g_1(r, -0)]\}$$

$$\sigma(r) = \{g_4(r, 0), g_3(r, 0)\}, \quad r > 1, \quad u(r) = 0 \tag{2.3}$$

and the Mellin transforms

$$u^{\circ+}(p) = \int_0^1 r^p u(r)\, dr, \quad \sigma^{\circ+}(p) = \int_0^1 r^p \sigma(r)\, dr, \quad \sigma^{\circ-}(p) = \int_1^{\infty} r^p \sigma(r)\, dr$$

as well a 2 x 2 matrix of the form

$$\begin{aligned}
2\{G\}_{11}(p) &= {}^1/_2[D(p, \theta_1)(\sin p\theta_1 \cos p\theta_1 + p \sin\theta_1 \cos\theta_1) + \\
&+ D(p, \theta_2)(\sin p\theta_2 \cos p\theta_2 + p \sin\theta_2 \cos\theta_2)] \\
2\{G\}_{12}(p) &= p(p-1)[\sin^2\theta_1 D(p, \theta_1) - \sin^2\theta_2 D(p, \theta_2)] \\
2\{G\}_{21}(p) &= -p(p+1)[\sin^2\theta_1 D(p, \theta_1) - \sin^2\theta_2 D(p, \theta_2)] \\
2\{G\}_{22}(p) &= {}^1/_2[D(p, \theta_1)(\sin p\theta_1 \cos p\theta_1 - p \sin\theta_1 \cos\theta_1) + \\
&+ D(p, \theta_2)(\sin p\theta_2 \cos p\theta_2 - p \sin\theta_2 \cos\theta_2)]
\end{aligned} \tag{2.4}$$

The vectors $u^{\circ+}(a)$, $\sigma^{\circ+}(p)$ are holomorphic in the half-plane $\delta > -{}^1/_2$, and the vector $\sigma^{\circ-}(p)$ in the halfplane $\delta < 0$. Therefore, a two-dimensional vector of the form

$$\varphi(p) = \begin{cases} u^{\circ+}(p) & (\delta \geqslant \delta_0) \\ \sigma^{\circ-}(p) & (\delta \leqslant \delta_0) \end{cases} \tag{2.5}$$

is piecewise-holomorphic with the jump lines L $\delta = \delta_0$ (δ_0 is any number from the open interval $(-{}^1/_2, 0)$). A two-dimensional vector of the form

$$\psi(p) = G(p)\, \sigma_0{}^{+}(p) \tag{2.6}$$

is the given vector. By virtue of (2.5) and (2.6) the equality (2.2) becomes

$$t \in L, \quad \varphi^{+}(t) = G(t)\, \varphi^{-}(t) + \psi(t) \tag{2.7}$$

on L, where now the limit values of the piecewise-holomorphic vector $\varphi(p)$ on the line of jumps to the right and left, respectively, correspond to the superscripts "plus" and "minus", in complete conformity with the notation in Sect. 1.

The equality (2.7) is a system of Wiener-Hopf equations, in vector-matrix form, for the unknown transforms of the derived displacements on the notch itself and for the stresses on its continuation; this is simultaneously an inhomogeneous Hilbert problem for the piecewise-holomorphic two-dimensional vector $\varphi(p)$ with the line of jumps L. We have

$$2t^{-1}\operatorname{dev} G(t) = \begin{Vmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{Vmatrix} \tag{2.8}$$

$$\Delta_{11} = {}^1/_2\,[D(t,\ \theta_1)\sin 2\theta_1 + D(t,\ \theta_2)\sin 2\theta_2]$$

$$\Delta_{12} = (t-1)\,[D(t,\ \theta_1)\sin^2\theta_1 - D(t,\ \theta_2)\sin^2\theta_2]$$

$$\Delta_{21} = -(t+1)\,[D(t,\ \theta_1)\sin^2\theta_1 - D(t,\ \theta_2)\sin^2\theta_2]$$

$$\Delta_{22} = -\Delta_{11}$$

Let us consider three cases of the representability of the matrix $G(t)$ in the form (1.3).

Case 1. The notch on the continuation of one of the sides of the wedge enveloping the half-plane ($\theta_2 = \pi$)

$$\operatorname{dev} G(t) = \frac{t\sin\theta_1 D(t,\theta_1)}{2}\begin{Vmatrix} \cos\theta_1 & (t-1)\sin\theta_1 \\ -(t+1)\sin\theta_1 & -\cos\theta_1 \end{Vmatrix} \tag{2.9}$$

Case 2. The notch emerging on the boundary of the half-plane ($\theta_1 + \theta_2 = \pi$)

$$\operatorname{dev} G(t) = \frac{t\sin\theta_1\,[D(t,\theta_1) - D(t,\theta_2)]}{2}\begin{Vmatrix} \cos\theta_1 & (t-1)\sin\theta_1 \\ -(t+1)\sin\theta_1 & -\cos\theta_1 \end{Vmatrix}. \tag{2.10}$$

Case 3. A semi-infinite slit with a breakpoint in the infinite plane ($\theta_1 + \theta_2 = 2\pi$); the expression $\operatorname{dev} G(t)$ retains the form (2.10).

As follows from (2.9), (2.10), the relationship

$$\operatorname{com} G(t) = B(t), \qquad B(p) = (l^2 + mn)^{-1/2}\begin{Vmatrix} l(p) & m(p) \\ n(p) & -l(p) \end{Vmatrix} \tag{2.11}$$

$$l(p) = \cos\theta_1, \qquad m(p) = (p-1)\sin\theta_1, \qquad n(p) = -(p+1)\sin\theta_1$$

holds in all the considered cases. The same notation $\Delta(t)$, $\varepsilon(t)$, $\lambda_{1,2}(t)$, $\varkappa_\Delta$, $\varkappa_\varepsilon$, as in Sect. 1, is retained for the remaining parameters of the matrix $G(t)$ The order of the polynomial $l^2 + mn = 1 - p^2\sin^2\theta_1$ equals $k = 2$, hence there are no conditions of the form (1.21).

According to (1.15), (1.19) and (1.20), the solution of the homogeneous problem (1.1) corresponding to problem (2.7), is

$$X^+(t)\,[X^-(t)]^{-1} = G(t)$$

$$\Lambda^{-1}(p)X(p) = I\operatorname{ch}[(1 - p^2\sin^2\theta_1)^{1/2}\,\beta(p)] + B(p)\operatorname{sh}[(1 - p^2\sin^2\theta_1)^{1/2}\,\beta(p)] \tag{2.12}$$

$$\Lambda(p) = (p - \delta_0)^{1/2}\exp\left[-\frac{1}{4\pi i}\int_L \frac{\ln\Delta(t)}{t-p}\,dt\right]$$

$$\beta(p) = -\frac{1}{2\pi i}\int_L \frac{(1 - t^2\sin^2\theta_1)^{-1/2}\,\varepsilon(t)}{t-p}\,dt$$

For $\delta_0 \to 0$, taking account of the symmetry (antisymmetry) of the separate functions relative to the argument τ on the imaginary axis, we obtain the following formulas:

$$\Lambda(p)\exp\left[-\frac{n}{2\pi}\int_0^\infty \frac{\ln|\Delta(i\tau)|}{\tau^2+p^2}d\tau\right] = \begin{cases} p^{1/2}, & -\frac{\pi}{2}\leqslant \arg p \leqslant \frac{\pi}{2}, \ \delta>0 \\ -ip^{1/2}, & \frac{\pi}{2}\leqslant \arg p\leqslant \frac{3\pi}{2}, \ \delta<0\end{cases}$$

$$\beta(p) = \frac{p}{\pi}\int_0^\infty \frac{(1+\tau^2\sin^2\theta_1)^{-1/2}\,\varepsilon(i\tau)}{\tau^2+p^2}d\tau \tag{2.13}$$

From (2.11), (2.12) and (2.13), we have directly for $\delta > 0$

$$\Lambda(p) = p\Lambda^{-1}(-p), \qquad \beta(p) = -\beta(-p), \qquad B(p) = B^{(*)}(-p)$$
$$X(p) = pX^{(*)(-1)}(-p) \tag{2.14}$$

The superscript $(*)$ denotes the transpose of the matrix. The following asymptotic dependencies, valid for large $|p|$, result from (2.13):

$$\beta(p) = q(p\sin\theta_1)^{-1}, \qquad q = \frac{\sin\theta_1}{\pi}\int_0^\infty (1+\tau^2\sin^2\theta_1)^{-1/2}\varepsilon(i\tau)\,d\tau$$

$$\begin{array}{lll} \delta>0, & \Lambda(p)\sim p^{1/2} & X(p)\sim p^{1/2}Q, \\ \delta<0, & \Lambda(p)\sim -ip^{1/2}, & X(p)\sim -ip^{1/2}Q, \end{array} \qquad Q = \begin{Vmatrix} \cos q & \sin q \\ -\sin q & \cos q\end{Vmatrix} \tag{2.15}$$

The matrix $X(p)$, defined by (2.12), (2.13) is the canonical solution of the homogeneous equation since it is piecewise-holomorphic together with the matrix $X^{-1}(p)$ (the line of jumps L is the imaginary axis), and the sum of the orders of the separate columns is 1.0, which agrees exactly with the order of the determinant. The branch of the piecewise-holomorphic matrix $X(p)$ defined in the right half-plane is analytically continuable into the left half-plane by using the matrix $(GX)(p)$ at least up to the vertical line $\delta = -1/2$; the inverse matrix $(X^{-1}G^{-1})(p)$ is hence also holomorphic. The pair of matrices $X(p)$ and $(GX)(p)$ obtained in this manner can be considered as limit values of the piecewise-holomorphic matrix $X(p)$ on the left and right on any vertical line L in the open strip $0 > \delta > -1/2$ Returning to the inhomogeneous problem (2.7), we obtain, by taking account of (2.6)

$$t\in L, \quad [X^+(t)]^{-1}\varphi(t) = [X^-(t)]^{-1}\varphi^-(t) + [X^+(t)]^{-1}\psi(t) \tag{2.16}$$
$$[X^+(t)]^{-1}\psi(t) = [X^-(t)]^{-1}\sigma^{\circ+}(t)$$

For large $|p|$ for loads bounded on the whole closed interval $0\leqslant r\leqslant 1$, $\sigma^{\circ+}(p)\sim$ $\sim O(p^{-1})$, hence the solution of problem (2.16) which vanishes at infinity is given by the formula

$$\varphi(p) = -\frac{X(p)}{2\pi i}\int_L \frac{[X^-(t)]^{-1}\sigma^{\circ+}(t)}{t-p}dt \tag{2.17}$$

The branch of the piecewise-holomorphic matrix $X(p)$ defined by (2.12), (2.13) in the left half-plane can be continued analytically into the right half-plane by using the matrix $(G^{-1}X)(p)$. However, the inverse matrix $(X^{-1}G)(p)$ will have a pole at the point $p = 0$ in this case. The results of calculations based on utilization of the first two dependencies (2.14) yield

$$\operatorname{Res}(\Delta^{-1/2}\Lambda)(0) = -\lim_{p\to 0}\sqrt{-p\Delta^{1/2}(p)} = -\lim_{p\to 0}\sqrt{-p(\lambda_1^{1/2}\lambda_2^{1/2})(p)}$$

$$\beta^+(0) = -\beta^-(0) = {}^1/_2\varepsilon(0) = {}^1/_4\lim_{p\to 0}\ln[(\lambda_1\lambda_2^{-1})(p)] \tag{2.18}$$

where $\lambda_{1,2}(p)$ are characteristic functions of the matrix $G(p)$. We obtain

$$-2\operatorname{Res}(X^{-1}G)(0)=\lim_{p\to 0}\{I\,[\sqrt{-p\lambda_1(p)}+\sqrt{-p\lambda_2(p)}]+ + H\,[\sqrt{-p\lambda_1(p)}-\sqrt{-p\lambda_2(p)}]\}$$

$$H=B(0),\quad \{H\}_{11}=\cos\theta_1,\quad \{H\}_{12}=-\sin\theta_1,\quad \{H_{21}\}=-\sin\theta_1,\quad \{H\}_{22}=-\cos\theta_1 \tag{2.19}$$

for the residue at the point $p=0$

3. Stress concentration on the continuation of the notch and the mutual displacement of its sides. Let us call a 2 x1 column vector of the form

$$n=\lim_{r\to 1+0}(r-1)^{1/2}\sigma(r) \tag{3.1}$$

the stress intensity vector. The components of the vector n are evidently coefficients of the intensity of the normal and tangential stresses on the continuation of the notch. We have ([6], p. 48) for large $|p|$

$$\sigma^{\circ-}(p)\sim i\Gamma(1/2)\,np^{-1/2} \tag{3.2}$$

where $\Gamma(x)$ is the standard notation for the Gamma function.
Comparing (2.17) and (3.2) we obtain by virtue of (2.5) and (2.15)

$$\sqrt{\pi}\,n=-\frac{Q}{2\pi i}\int_L[X^-(t)]^{-1}\sigma^{\circ+}(t)\,dt \tag{3.3}$$

Substituting the expression for the transform $\sigma^{\circ+}(t)$ in terms of its original into (3.3), we have

$$-\sqrt{\pi}\,n=\frac{Q}{2\pi i}\int_L[X^-(t)]^{-1}dt\int_0^1 r^t\sigma(r)\,dr \tag{3.4}$$

Inverting the order of integration in (3.4), we obtain

$$n=\int_0^1 N(r_0)\,\sigma(r_0)\,dr_0,\qquad -\sqrt{\pi}\,N(r_0)=QM(r_0) \tag{3.5}$$

$$M(r_0)=\frac{1}{2\pi i}\int_L r_0{}^t[X^-(t)]^{-1},\quad dt=\frac{1}{2\pi i}\int_L r_0{}^t[X^+(t)]^{-1}G(t)\,dt \tag{3.6}$$

Here $N(r_0)$ and $M(r_0)$ are 2x2 matrices. The former is the matrix Green function for the intensity vector since its magnitude is given by the product of the matrix $N(r_0)$ by a column-vector on the right under the effect of equal and opposite forces at the point r_0, where the vector components are common for both the normal and tangential force components.

Let us now introduce the 2x1 column vector $v(r)$ of the mutual displacement of points on opposit sides of the notch and at identical distances from the wedge vertex, as well as the corresponding vector Mellin transform

$$v(r)=\{[u_\varphi(r,+0)-u_\varphi(r,-0)],\qquad [u_r(r,+0)-u_r(r,-0)]\}$$

$$r>1,\quad v(r)=0,\qquad v^\circ(p)=\int_0^1 r^p v(r)\,dr$$

We have

$$1/4\,Ev'(r)=u(r),\qquad 1/4\,Ev^\circ(p)=-(p+1)^{-1}u^\circ(p+1)$$

Taking account of (2.14), we obtain from (2.17)

$$ {}^{1}/_{4}Ev^{\circ}(p) = \frac{X^{(*)(-1)}(-p-1)}{2\pi i}\int\limits_{L}\frac{[X^{-}(t)]^{-1}\sigma^{\circ+}(t)}{t-p-1}\,dt \tag{3.7} $$

By the theorem of multiplication of transforms ([7], p. 503), the original of the left side in (3.7) is given by the formula

$$ {}^{1}/_{4}Ev(r) = \int\limits_{0}^{\infty} M^{(*)}(r\xi)\,\eta(\xi)\,d\xi $$

$$ \eta^{\circ}(p) = \int\limits_{0}^{\infty} r^{p}\eta(r)\,dr = \frac{1}{2\pi i}\int\limits_{L}\frac{[X^{-}(t)]^{-1}\sigma^{\circ+}(t)}{t+p}\,dt \tag{3.8} $$

where $\eta(r)$ is also a 2x1 column vector. The theorem of multiplication of originals ([7], p. 505) for the form of the Mellin transform taken in herein yields

$$ \eta(r) = \eta_{0}(r)\begin{cases} 0 & (r \leqslant 1) \\ -\dfrac{1}{r} & (r > 1) \end{cases} \tag{3.9} $$

$$ \eta_{0}^{\bullet}(p) = \int\limits_{0}^{\infty} r^{p}\eta_{0}(r)\,dr = [X(p)]^{-1}\sigma^{\circ+}(p) $$

Again applying the theorem of multiplication of transforms, we have

$$ \eta_{0}(r) = \int\limits_{0}^{1} M(r_{0}r)\,\sigma(r_{0})\,dr_{0} \tag{3.10} $$

As a set, (3.8) - (3.10) yield

$$ \frac{1}{4}Ev(r) = -\int\limits_{1}^{\infty} M^{(*)}(r\xi)\frac{d\xi}{\xi}\int\limits_{0}^{1} M(r_{0}\xi)\,\sigma(r_{0})\,dr_{0} \tag{3.11} $$

Inverting the order of integration in (3.11), we obtain

$$ v(r) = \int\limits_{0}^{1} V(r, r_{0})\,\sigma(r_{0})\,dr_{0} \tag{3.12} $$

$$ \frac{1}{4}EV(r, r_{0}) = -\int\limits_{1}^{\infty} M^{(*)}(r\xi)\,M(r_{0}\xi)\frac{d\xi}{\xi} $$

Here $V(r, r_0)$ is a 2x2 matrix, which is the matrix Green function for the mutually displacements of the sides of the notch. Indeed, the column-vector $v(r)$ of the mutual displacements of the sides at the point r is given by the product of the matrix $V(r, r_0)$ by a column-vector on the right, under the effect of equal and opposite concentrated forces at the point r_0, where the vector components are common for both normal and tangential force components. Taking into account that the matrix $M^{\cdot}(r_0)$ is a zero matrix when the value of the argument exceeds 1.0, we obtain

$$ \frac{1}{4}EV(r, r_{0}) = -\int\limits_{1}^{\vartheta}\xi^{-1}M^{(*)}(r\xi)\,M(r_{0}\xi)\,d\xi = -\int\limits_{\theta}^{1}x^{-1}M^{(*)}(r/x)\,M(r_{0}/x)\,dx $$

$$ (\vartheta = \inf(1/r, 1/r_{0}), \quad \theta = \sup(r, r_{0})) \tag{3.13} $$

The relation between the matrix Green functions for the intensity vector and the vector of the mutual displacements of the sides under the effect of equal and opposite concentrated forces is given by (3.5), (3.12) and (3.13).

The relationship

$$V(r, r_0) = V^{(*)}(r_0, r) \tag{3.14}$$

results also from (3.13). The equality (3.14) expresses the law of reciprocity of the displacements under the conditions of this problem.

Turning to the construction of approximate formulas to calculate the matrix function $M(r_0)$, we first present two identities which result directly from the Jordan lemma and the theorem of residues

$$\frac{1}{2\pi i}\int_L \frac{\Gamma(-t)\,r_0^{\,t}}{\Gamma(1/2-t)}\,dt = -\sum_{k=0}^{\infty}\frac{r_0^{\,k}\operatorname{Res}\Gamma(-k)}{\Gamma(1/2-k)} = \sum_{k=0}^{\infty}\frac{(-1)^k r_0^{\,k}}{k!\,\Gamma(1/2-k)} =$$

$$= \frac{1}{\pi}\sum_{k=0}^{\infty}\frac{r_0^{\,k}\Gamma(k+1/2)}{\Gamma(k+1)} = \frac{1}{\sqrt{\pi}}\left(1+\sum_{k=1}^{\infty} r_0^{\,k}\prod_{j=1}^{k}\frac{j-1/2}{j}\right) = \frac{1}{\sqrt{\pi}}(1-r_0)^{-1/2} \tag{3.15}$$

$$\frac{1}{2\pi i}\int_L \frac{\Gamma(-2\theta t/\pi)(\cos 2\theta t-1)\,r_0^{\,t}\,dt}{\Gamma(1/2-2\theta t/\pi)\cos 2\theta t} = \sum_{k=1}^{\infty}\frac{r_0^{\pi k/2\theta}(1-\cos\pi k)}{\cos\pi k\,\Gamma(1/2-k)}\operatorname{Res}\Gamma\left(-\frac{2\theta}{\pi}\frac{k\pi}{2\theta}\right) =$$

$$= \frac{1}{\theta}\sum_{k=1,3,5\ldots}^{\infty}\frac{\Gamma(k+1/2)\,r_0^{\pi k/2\theta}}{\Gamma(k+1)} = \frac{\sqrt{\pi}}{2\theta}\left[(1-r_0^{\pi/2\theta})^{-1/2}+(1+r_0^{\pi/2\theta})^{-1/2}\right] \tag{3.16}$$

In the former of the cases considered (a notch on the continuation of one of the sides of the wedge, $\theta_2=\pi$) we write (3.6) as

$$M(r_0) = 1/2\left\{\left(\frac{1}{\pi}\right)^{1/2}(1-r_0)^{-1/2}+\left(\frac{1}{2\theta_1}\right)^{1/2}\left[(1-r_0^{\pi/2\theta_1})^{-1/2}-(1+r_0^{\pi/2\theta_1})^{-1/2}\right]\right\}Q^{-1}+$$

$$+\frac{1}{2\pi i}\int_L r_0^{\,t}\left\{[X^+(t)]^{-1}G(t)-1/2\left[\frac{\Gamma(-2\theta_1 t/\pi)(\cos 2\theta_1 t-1)}{\Gamma(1/2-2\theta_1 t/\pi)\cos 2\theta_1 t}\left(\frac{2\theta_1}{\pi}\right)^{1/2}+\frac{\Gamma(-t)}{\Gamma(1/2-t)}\right]Q^{-1}\right\}dt \tag{3.17}$$

In the two remaining cases ($\theta_1+\theta_2=\pi$, $\theta_1+\theta_2=2\pi$) we write

$$M(r_0) = 1/2\left\{\left(\frac{1}{2\theta_1}\right)^{1/2}\left[(1-r_0^{\pi/2\theta_1})^{-1/2}-(1+r_0^{\pi/2\theta_1})^{-1/2}\right]+\right.$$

$$\left.+\left(\frac{1}{2\theta_2}\right)^{1/2}\left[(1-r_0^{\pi/2\theta_2})^{-1/2}-(1+r_0^{\pi/2\theta_2})^{-1/2}\right]\right\}Q^{-1}+\frac{1}{2\pi i}\int_L r_0^{\,t}\left\{[X^+(t)]^{-1}G(t)-\right.$$

$$\left.-\frac{1}{2}\left[\left(\frac{2\theta_1}{\pi}\right)^{1/2}\frac{\Gamma(-2\theta_1 t/\pi)(\cos 2\theta_1 t-1)}{\Gamma(1/2-2\theta_1 t/\pi)\cos 2\theta_1 t}+\left(\frac{2\theta_2}{\pi}\right)^{1/2}\frac{\Gamma(-2\theta_2 t/\pi)(\cos 2\theta_2 t-1)}{\Gamma(1/2-2\theta_2 t/\pi)\cos 2\theta_2 t}\right]Q^{-1}\right\}dt \tag{3.18}$$

Let us consider a matrix defined in the half-plane $\delta > -1/2$ whose boundary value on L is the integrand in (3.18). It has a pole at the points $s\pi/2\theta_1$, $s\pi/2\theta_2$ (s is an integer) and also at the poles of the functions $D(p, \theta_1)$ and $D(p, \theta_2)$, which have the respective form for large s

$$2\theta_{1,2}p_{2s}/\pi \sim (2s+1)+i(2/\pi)\ln[(2s+1)\pi\theta_{1,2}^{-1}\sin\theta_{1,2}], \qquad p_{2s+1}=\bar{p}_{2s}$$

Let us agree to understand a set of pairs of complex-conjugate poles of the functions $D(p, \theta_1)$ and $D(p, \theta_2)$, as well as the points $s\pi/2\theta_1$ and $s\pi/2\theta_2$ as the group of poles with number s. Combining components referring to a certain group of poles into one term each time in the decomposition of the integral (3.18) in residues, we obtain a rapidly converging series, and a completely analogous situation in the decomposition of the integral (3.17). We obtain approximate expressions for the matrix $M(r_0)$ which are suitable for utilization in the whole interval $0 \leqslant r_0 \leqslant 1$, by retaining only terms corresponding to the first two poles $p=0$, $p=1$ in the expansions of the integrals in residues of the integrands. From (3.17) and (3.18) we have

$$M(r_0) \approx M_1(r_0) \tag{3.19}$$

$$\theta_2 = \pi, \quad M_1(r_0) = -\operatorname{Res}(X^{-1}G)(0) - r_0 \operatorname{Res}(X^{-1}G)(1) + {}^1/_2 \{(2\theta_1)^{-1/2} [(1 - r_0^{\pi/2\theta_1})^{-1/2} - (1 + r_0^{\pi/2\theta_1})^{-1/2} - r_0^{\pi/2\theta_1}] + \pi^{-1/2} [(1 - r_0)^{-1/2} - 1 - {}^1/_2 r_0]\} Q^{-1}$$

$$\theta_1 + \theta_2 = \pi, \ \theta_1 + \theta_2 = 2\pi, \ M_1(r_0) = -\operatorname{Res}(X^{-1}G)(0) - r_0 \operatorname{Res}(X^{-1}G)(1) + {}^1/_2 \{(2\theta_1)^{-1/2} [(1 - r_0^{\pi/2\theta_1})^{-1/2} - (1 + r_0^{\pi/2\theta_1})^{-1/2} - r_0^{\pi/2\theta_1}] + (2\theta_2)^{-1/2} [(1 - r_0^{\pi/2\theta_2})^{1/2} - (1 + r_0^{\pi/2\theta_2})^{-1/2} - r_0^{\pi/2\theta_2}]\} Q^{-1}$$

where $M(r_0)$ is a 2x2 matrix. The quantity $\operatorname{Res}(X^{-1}G)(1)$ is calculated by (2.4), (2.12) (2.13). Utilizing the notation

$$\Phi(x, \theta) = [(1 - x^{\pi/2\theta})^{-1/2} - (1 + x^{\pi/2\theta})^{-1/2} - x^{\pi/2\theta}] (2\theta)^{-1/2}$$

we obtain the formulas presented below.

Case 1. Notch on the continuation of one of the sides of a wedge enveloping a half-plane ($\theta_2 = \pi$)

$$M_1(r_0) = A_0 - r_0 A_1 + {}^1/_2 \{\Phi(r_0, \theta_1) + \pi^{-1/2} [(1 - r_0)^{-1/2} - 1 - {}^1/_2 r_0]\} Q^{-1} \tag{3.20}$$

$$\sqrt{2} A_0 = \begin{Vmatrix} \sqrt{a_1} \cos^2 \vartheta + \sqrt{a_2} \sin^2 \vartheta & -(\sqrt{a_1} - \sqrt{a_2}) \sin \vartheta \cos \vartheta \\ -(\sqrt{a_1} - \sqrt{a_2}) \sin \vartheta \cos \vartheta & \sqrt{a_1} \sin^2 \vartheta + \sqrt{a_2} \cos^2 \vartheta \end{Vmatrix}$$

$$2\Lambda(1) A_1 = \frac{e^{-\tau}}{\operatorname{tg} \theta_1 - \theta_1} \begin{Vmatrix} 1 & 0 \\ -\operatorname{tg} \theta_1 & 0 \end{Vmatrix} - \frac{1}{\pi} \begin{Vmatrix} e^{-\tau} & 0 \\ 2 \operatorname{tg} \theta_1 \operatorname{sh} \tau & e^{\tau} \end{Vmatrix}$$

$$a_{1,2} = \frac{1}{\theta_1 \mp \sin \theta_1} + \frac{1}{\pi}, \qquad \vartheta = \frac{\theta_1}{2}, \qquad \tau = \cos \theta_1 \beta(1)$$

Values of the quantities q, as well as of the nonzero elements of the matrix A_1 for values of the angle $\theta_1 = 15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165°$ have been calculated on the "Ural-2" computer and are presented in Table 1.

Case 2. Notch emerging on the boundary of the half-plane ($\theta_1 + \theta_2 = \pi$)

$$M_1(r_0) = B_0 - r_0 B_1 + {}^1/_2 [\Phi(r_0, \theta_1) + \Phi(r_0, \pi - \theta_1)] Q^{-1} \tag{3.21}$$

$$\sqrt{2} B_0 = \begin{Vmatrix} \sqrt{b_1} \cos^2 \vartheta + \sqrt{b_2} \sin^2 \vartheta & -(\sqrt{b_1} - \sqrt{b_2}) \sin \vartheta \cos \vartheta \\ -(\sqrt{b_1} - \sqrt{b_2}) \sin \vartheta \cos \vartheta & \sqrt{b_1} \sin^2 \vartheta + \sqrt{b_2} \cos^2 \vartheta \end{Vmatrix}$$

$$2\Lambda(1) B_1 = e^{-\tau} \left(\frac{1}{\operatorname{tg} \theta_1 - \theta_1} - \frac{1}{\operatorname{tg} \theta_1 - \theta_1 + \pi} \right) \begin{Vmatrix} 1 & 0 \\ -\operatorname{tg} \theta_1 & 0 \end{Vmatrix}$$

$$b_{1,2} = \frac{\pi}{\theta_1 (\pi - \theta_1) - \sin^2 \theta_1 \mp (\pi - 2\theta_1) \sin \theta_1}, \qquad \vartheta = \frac{\theta_1}{2}, \qquad \tau = \cos \theta_1 \beta(1)$$

Values of the quantities q and nonzero elements of the matrix B_1 for values of the angle $\theta_1 = 15, 30, 45, 60, 75°$ are presented in Table 1. In the case of a notch on the bisectrix of the half-plane ($\theta_1 = 90°$) B_1 is a zero matrix, and $q = 0$. For values of the angle θ_1 in the 90—180° range, elements of the matrix B_1 can be expresses in terms of values of the corresponding elements in the 0—90° range since the diagonal elements are symmetric, and those outside the diagonal, are antisymmetric functions of the argument $\theta_1 - 90°$.

Case 3. Semi-infinite slit with breakpoint in the infinite plane ($\theta_1 + \theta_2 = 2\pi$)

$$M_1(r_0) = C_0 - r_0 C_1 + {}^1/_2 [\Phi(r_0, \theta_1) + \Phi(r_0, 2\pi - \theta_1)] Q^{-1} \tag{3.22}$$

$$\sqrt{2} C_0 = \left\| \begin{matrix} \sqrt{c_1}\cos^2\vartheta + \sqrt{c_2}\sin^2\vartheta & -(\sqrt{c_1} - \sqrt{c_2})\sin\vartheta\cos\vartheta \\ -(\sqrt{c_1} - \sqrt{c_2})\sin\vartheta\cos\vartheta & \sqrt{c_1}\sin^2\vartheta + \sqrt{c_2}\cos^2\vartheta \end{matrix} \right\|$$

$$2\Lambda(1) C_1 = e^{-\tau}\left(\frac{1}{\operatorname{tg}\theta_1 - \theta_1} - \frac{1}{\operatorname{tg}\theta_1 - \theta_1 + 2\pi}\right) \left\| \begin{matrix} 1 & 0 \\ -\operatorname{tg}\theta_1 & 0 \end{matrix} \right\|$$

$$c_{1,2} = \frac{2\pi}{\theta_1(2\pi - \theta_1) - \sin^2\theta_1 \mp (2\pi - 2\theta_1)\sin\theta_1}, \quad \vartheta = \frac{\theta_1}{2}, \quad \tau = \cos\theta_1\, \beta(1)$$

Values of the quantities q, as well as the nonzero elements of the matrix C_1 are presented in Table 1 for values of the angle $\theta_1 = 105, 120, 135, 150, 165°$. In the case when the value of the angle between the semi-infinite slit and the finite section from the breakpoint exceeds 75°, the error in the approximate formula (3.19) is quite significant. This is connected with the presence of poles of the function $D(p, \theta_2)$ near the value $p = 1$ (and even closer to the origin for $\theta_1 \leqslant 103°$) discarded in the derivation. Meanwhile, for small values of the angle θ_1 it is necessary to verify the presence of an unopened semi-infinite notch, which it is difficult to realize in practice.

In conclusion, let us note that the problem of elastic equilibrium of a wedge with a notch on the bisectrix reduces to one functional Wiener-Hopf equation and hence, it always has a solution in the form of Cauchy type integrals. This problem was considered in [8, 9, 10, 11], for example, for distributed loads.

Table 1.

$\theta_1°$	q	$(A_1)_{11}$	$(A_1)_{21}$	$(A_1)_{22}$
$\theta_2 = \pi$				
15	0.536	10.732	−2.903	−0.103
30	0.423	2.861	−1.720	−0.113
45	0.324	1.046	−1.163	−0.117
60	0.239	0.353	−0.820	−0.120
75	0.169	0.034	−0.586	−0.123
90	0.113	−0.126	−0.420	−0.126
105	0.071	−0.209	−0.296	−0.129
120	0.040	−0.251	−0.205	−0.132
135	0.020	−0.271	−0.136	−0.135
150	0.008	−0.279	−0.082	−0.138
165	0.002	−0.282	−0.038	−0.140

$\theta_1°$	q	$(B_1)_{11}$	$(B_1)_{21}$
$\theta_1 + \theta_2 = \pi$			
15	0.536	10.786	−2.890
30	0.419	2.892	−1.669
45	0.308	1.056	−1.056
60	0.203	0.368	−0.637
75	0.101	0.081	−0.302
$\theta_1°$	q	$(C_1)_{11}$	$(C_1)_{21}$
$\theta_1 + \theta_2 = 2\pi$			
105	0.068	−0.672	−2.506
120	0.036	−0.295	−0.510
135	0.016	−0.286	−0.286
150	0.005	−0.285	−0.165
165	0.001	−0.284	−0.077

BIBLIOGRAPHY

1. Chebotarev, G. N., On the closed form solution of the Riemann boundary value problem for a system of n pairs of functions. Uch. Zap. Kazan' Univ. Vol. 116, No. 4, 1956.

2. Muskhelishvili, N. I., Singular Integral Equations, 3d Ed., Moscow, FIZMATGIZ, 1968.

3. Vekua, N. P., Systems of Singular Integral Equations and Some Boundary Value Problems. 2d Ed., Moscow, "NAUKA" 1970.
4. Gakhov, F. D., Riemann boundary value problem for systems of n pairs of functions. Usp. Matem. Nauk, Vol. 7, No. 4, 1952.
5. Ufliand, Ia. S., Integral Transforms in Elasticity Theory. 2d Ed., Leningrad, "NAUKA", 1968.
6. Noble, B., Methods Based on the Wiener-Hopf Technique for the Solution of Partial Differential Equations. Pergamon Press, 1959.
7. Lavrent'ev, M. A. and Shabat, B. V., Methods of Complex Variable Function Theory. 3d Ed. Moscow, "NAUKA", 1965.
8. Srivastav R. P. and Prem Narain, Certain two-dimensional problems of stress distribution in wedge-shaped elastic solids under discontinuous load. Proc. Cambr. Phil. Soc., Vol. 61, No. 4, 1965.
9. Bantsuri, R. D., Solution of the first fundamental problem of elasticity theory for a wedge having a finite slit. Dokl. Akad. Nauk SSSR, Vol. 167, No. 6, 1966.
10. Smetanin, B. I., Some problems of slits in an elastic wedge and layer. Inzh. Zh., Mekh. Tverd. Tela, No. 2, 1968.
11. Smetanin, B. I., On a mixed problem of elasticity theory for a wedge. PMM, Vol. 32, No. 4, 1968.

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ALGORITHM OF THE METHOD OF CHARACTERISTICS FOR THE ANALYSIS OF NONLINEAR ONE-DIMENSIONAL WAVE PROCESSES OF CONICAL AND CYLINDRICAL SHELL DEFORMATION

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An algorithm is proposed for realizing the method of characteristics for the analysis of one-dimensional wave processes excited by the edge effect and described by a quasilinear system of differential equations having several pairs of families of characteristics. The algorithm is written in Lagrangian coordinates for conical and cylindrical shells, on the basis of a quasilinear system of sixth order equations of a geometrically nonlinear theory of Timoshenko type [1].

The algorithm presumes the absence of strong discontinuities, i.e., of discontinuities in the first derivatives of the shell displacement, which will limit the class of admissible edge effects and permit carrying out the analysis up to the appearance of the first shock in problems where the shocks originate during wave propagation. Despite this, the proposed algorithm permits elucidation of specific properties of the wave solution in nonlinear theory. An illustrative example is given for a conical shell.

In speaking of the one-dimensional transients of conical and cylindrical shell deformation, we shall have in mind the axisymmetric processes of these objects